Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Linear Algebra-II

Subject Code: 4SC04LIA1/4SC04MTC2 Branch: B.Sc. (Mathematics, Physics)

Semester: 4 Date :01/05/2018 Time : 10:30 To 01:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Inner product space **(01)**

b) Find angle between $(2,-1,3)$ and $(3,0,2)$. **(01)**

c) Define:Linear transformation **(01)**

d) True/false: Every vector space is inner product space. **(01)**

e) Define: Orthonormal basis **(01)**

f) If $T: R^2 \rightarrow R^2$ is orthogonal then find $\|Tx\|$ where $x = (2,3)$. **(01)**

g) State Cauchy-Swartz inequality for norm. **(01)**

h) True/false: Every inner product space is norm linear space. **(01)**

i) What do you mean by Conics and Quadrics ? **(01)**

j) True/False: Constant map is linear transformation. **(01)**

k) Write orthonormal basis of M_{22} . **(01)**

l) True/False: If $\langle u, v\rangle = 0$ then $u \perp v$. **(01)**

m) If T is linear transformation then show that $T(0) = 0$. **(01)**

n) If $u = (5, -4, 0)$ then find unit vector along u . **(01)**

Attempt any four questions from Q-2 to Q-8.

Q-2 **Attempt all questions** **(14)**

a) Apply Gram-Schmidt process to obtain orthonormal set **(07)**

$$\left\{\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 5 & 4 \end{bmatrix}\right\} \text{in } M_{22}.$$

b) Show that the medians of triangle are concurrent. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Using gram-Schmidt Orthogonalization process find orthogonal set from the set$\{(1,0,\ 2),(0,1,\ 1),(-1,\ 1,3)\}$. **(07)**

b) Prove that a parallelogram is rectangle iff the diagonal are of equal length. **(05)**

c) Prove that if A is orthogonal matrix then $\det(A) = \pm 1$. **(02)**

Q-4 **Attempt all questions** **(14)**

a) State and prove Riesz-representation theorem. **(08)**

b) If$x\ \perp y$ and $x\ \perp z$ then show that$x\ \perp(\alpha\, y+\beta\, z)\ \forall\ \alpha,\beta\in R$. **(03)**

c) Show that every orthogonal set is linearly independent but converse need not be true. **(03)**

Q-5 **Attempt all questions** **(14)**

a) Find determinant of $\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 0 & -4 & 0 \\ 5 & 7 & 8 & 5 \\ 0 & 2 & 2 & -9 \end{bmatrix}$ by using definition. **(07)**

b) With usual notation and figure show that$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$and $\rho_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$. **(07)**

Q-6 **Attempt all questions** **(14)**

a) Solve the system of equation by Cramer's rule $3x+y=0, 2y\ +\ z\ = 1$ and $x\ +\ 5z\ =\ 2$. **(07)**

b) Show that a linear transformation T from R^n to R^n is orthogonal iff the vectors$T(e_1),T(e_2),\ldots,T(e_n)$form an orthonormal basis of R^n,where $e_1,e_2,$ $\ldots.e_n$areorthonormal basis of R^n. **(07)**

Q-7 **Attempt all questions** **(14)**

a) Show that the matrix **(06)**

$$A = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$ is orthogonal matrix.

b) Show that $det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$. **(04)**

c) Find the Eigen values of $= \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$. **(04)**

Q-8 **Attempt all questions** **(14)**

a) State and Caley-Hamilton theorem and verify it for A= $\begin{pmatrix} 3 & 0 & 1 \\ -1 & 2 & 2 \\ 2 & 0 & 3 \end{pmatrix}$ also findA^{-3}if possible . **(07)**

b) Reduce the equation$3x^2+2xy+4yz+2xz-2x-14y-2z=0$ into standard form. **(07)**